ASSIGNMENT MANUAL OF ALGEBRA

SAMPSON

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This Assignment Manual can be used only in conjunction with ELEMENTARY ALGEBRA (FIRST COURSE) by Stone and Millis which is published by BENJ. H. SANBORN & Co.

This Assignment Manual had its origin because of the apparent need for such a text in connection with the supervised study of algebra as conducted at the Huntington School.

It should be suited to any algebra class where the Stone and Millis Elementary Algebra (First Course) has been adopted.

PREFACE

The difficulties generally encountered by algebra students seem to be principally due to study without *real* supervision. Few students of the age of those pursuing algebra know how to study or just what to study in the lesson assigned. The object of this manual is to remedy that condition.

To use this text effectively, the period should be divided into two parts—laboratory and recitation. The assignments of this manual should be studied and discussed during the laboratory period. This prepares for the recitation period to follow, during which problems of a similar nature should be solved without reference to any text. In brief, the idea is to prepare properly during the study or laboratory period for the actual work to be accomplished during the recitation. The process may be reversed (recitation first and laboratory second) if the subject or conditions favor such an arrangement.

If the manual is used in evening classes, each assignment outlines as much work as the evening student can generally do for a lesson outside of class. He has a definite assignment and comes to recitation prepared to ask questions more intelligently than would otherwise be the case.

Another advantage obtained by using this text is that both teacher and student know at all times what the work for each day is to be. A rule should be made at the start that the assignment for a certain day must be covered on that day. If this is adhered to there will be no doubt at any time as to whether or not the course is to be properly covered during the year. The objectionable features of hurrying at the end

are eliminated. The end of the year finds the work accomplished without haste, or waste of time and effort.

The number of assignments (150) has been selected because the author feels that it is not necessary to devote more than thirty full weeks of the year to advanced work. This leaves ample margin for frequent tests and review and any irregularities that may occur.

The number of problems given represents a minimum rather than a maximum. Problems whose solution is not required may be solved during the recitation period or assigned for work outside of class.

The study problem is a real problem which must be solved if the boys and girls in our schools are to secure the best results from the work pursued. Does this manual help in any way to solve that problem as far as the study of elementary algebra is concerned? Comments, suggestions and criticisms are desired.

CHARLES H. SAMPSON.

September, 1917.

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(*The chapter numbers are those of the Stone-Millis Elementary Algebra, First Course.*)

CHAPTER I

THE FORMULA: GENERAL NUMBER

ASSIGNMENT 1

Come to class prepared to answer the following arithmetical questions:

1. What are the signs of addition, subtraction, multiplication and division? Explain their meaning.

2. In what different ways may the process of multiplication be indicated?

3. In what different ways may the process of division be indicated?

4. When we write $2b$, a process of multiplication is indicated. When we write 24, does this mean that 2 is multiplied by 4? What is the difference?

For this assignment solve the following problems on pages 3 and 4:

1, 6, 10, 12, 14, 17, 20, 24, 26, 27, 30, and 37.

Most of these problems require the application of the process of **substitution.** This means that something is put in place of (substituted for) something else. If $w = ab$ and $a = 2$ and $b = 3$, the value of w would be 2 times 3, because in place of a and b their numerical values have been substituted.

Bear in mind at all times that a strong similarity exists between the fundamental rules of algebra and arithmetic.

ASSIGNMENT 2

Study carefully §5 on page 5 and apply the principles learned to problems 1, 2, 4, and 6 on the same page.

Study sections 6 and 7 and apply the principles to problems 7, 9, 11, 13, and 19 on page 7.

Problem 8 is worked here to illustrate the general process.

Find the value of $(g + 10) \div (5 - g)$ when $g = 3$.

In place of g, put 3.

We then have, $(3 + 10) \div (5 - 3)$.

This equals $13 \div 2$, the result being $6\frac{1}{2}$.

In the formula $\sqrt{z^2 - y^2}$ if z equalled 50 and y, 30,

$\sqrt{z^2 - y^2}$ would be expressed $\sqrt{(50)^2 - (30)^2}$.

$\sqrt{(50)^2 - (30)^2} = \sqrt{2500 - 900} = \sqrt{1600} = 40$.

Problems 18 to 23 inclusive are solved in this manner.

ASSIGNMENT 3

Study sections 8, 9 and 10 on page 8 and 11 on page 9. Go carefully through exercises 1—20 beginning on page 8 and express mentally the answers to the questions indicated. Solve exercises 1, 3 and 6 on page 9, and exercises 8, 15, and 22 on page 10.

The method of solution here is similar to that of the preceding exercise but somewhat more difficult. Exercise 12 is here solved. Study the solution.

$$v = \tfrac{1}{3}h\,(b + \mathrm{B} + \sqrt{bB}).$$

If $h = 6$ inches, $b = 16$ square inches, and $B = 49$ square inches, what is the value of v?

Substituting these values in the formula, we have

$$V = \tfrac{6}{3}\,(16 + 49 + \sqrt{16 \times 49}).$$
$$V = 2\,(16 + 49 + (4 \times 7)).$$
$$V = 2\,(16 + 49 + 28).$$
$$V = 186.$$

ASSIGNMENT 4

Study carefully sections 13 and 14 and solve mentally every other problem from 1 to 28 inclusive (page 11) beginning with 1.

Solve exercises 29, 33, and 39, remembering as you solve them that only similar terms can be combined.

Under "Supplementary Exercises", page 12, solve problems 6, 11, 17, 24, and 28. These are somewhat in the nature of a review and every effort should be made to secure the correct solution.

CHAPTER II

THE EQUATION

ASSIGNMENT 5

Study very carefully sections 15, 16, 17, 18, and 19. The solution of equations is an important feature of algebra study and should be thoroughly understood.

The important thing to remember about an equation is that the right side and the left side always have the same value, even though there may be several terms on one side of the equality sign and only one term on the other. You have no right to place an equality sign between two quantities which are not equal.

Solve the following problems: 1, 5, and 10 under "Oral Exercises", page 20; 1 and 4 under "Written Exercises", page 21; 25, 29, and 31 under "Oral Exercises", page 24; and 7, 9, and 13 under "Written Exercises", page 24.

ASSIGNMENT 6

Study §20 thoroughly, giving especial attention to the rules on page 26. After these are understood, solve the following exercises: 4, 8, 15, 18, 24, 29, and 30.

ASSIGNMENT 7

The solution of literal equations requires thought and concentration. You are cautioned to think exactly what is stated in words before you attempt to write the algebraic equivalent of it. Read the statements in words so carefully that you shall be able to make these statements before writing the algebraic statement.

There are four things to do in solving a problem. These are:

(1) Let x equal what is to be found out.

(2) Establish a relation between every number in the problem and x, by means of process signs.

(3) State algebraically exactly what the problem states in words.

(4) Solve the equation.

Beginning on page 28, solve exercises 1, 5, 8, 12, 16, 23, 29, and 32.

CHAPTER III

POSITIVE AND NEGATIVE NUMBERS

ASSIGNMENT 8

Study carefully sections 22, 23, 24, 25, 26, and 27. Before you proceed with the solution of exercises be absolutely sure that you understand what positive and negative numbers are.

Solve mentally the ten exercises on page 36. In addition, solve, in §27, exercises 1, 5, 8, 12, 18, and 21. In §28 solve exercises 1, 4, and 6. In §29 solve exercises 2 and 4.

ASSIGNMENT 9

Study carefully §30, devoting especial attention to rules (1) and (2) on page 43. Understand particularly what is

meant by the absolute value and what sign is to be used in the result. These two rules are very important and must be understood to solve satisfactorily problems involving the addition of positive and negative numbers.

Solve the following exercises beginning on page 44:

1, 3, 7, 12, 15, first two in exercise 21, first two in exercise 22, 29, 33, and 38.

ASSIGNMENT 10

Study §31. The important thing to remember is the rule given on page 48. Just think that the sign of the subtrahend is to be changed and then proceed as in addition.

Beginning on page 48, solve exercises 2, 3, 6, 10, 12, 18, the first two problems of exercise 19, the first two of exercises 20, 25, and 30.

When solving exercise 18 remember that a minus times a minus gives a plus and a minus times a plus gives a minus.

$$2 - (-5) = 2 + 5 = 7. \qquad -5 - (+3) = -5 - 3 = -8. \qquad +6 - (-2) = +6 + 2 = +8.$$

ASSIGNMENT 11

Study the rule under §32, page 50, and the three rules given on page 51. These are unusually important and must be understood if multiplication problems are to be satisfactorily solved.

3×5 is the same as $+5 + 5 + 5 = +15$.

$(+3) \times (-5)$ is the same as $(-5) + (-5) + (-5) = -15$.

$(+5) \times (-3)$ is the same as $(-3) + (-3) + (-3) + (-3) + (-3) = -15$.

When two numbers having like signs are multiplied the sign of the product is always plus.

When two numbers having unlike signs are multiplied the sign of the product is always minus.

The proper method of dealing with exponents is often confusing to the algebra student. Every number has an exponent. If it is not expressed it is understood to be 1; thus c is the same as c^1. It is not customary to write the exponent when it is 1, but it must not be overlooked. Note the following:

$$3 \times 3 \times 3 = 3^1 \times 3^1 \times 3^1 = 3^{1+1+1} = 3^3 = 27$$

Solve the following problems beginning on page 53: 2, 6, 10, 12, 14, 19, 27, 30, 33, 36, and 38.

ASSIGNMENT 12

Study the rules (1) and (2) given under §35. As you study division think of it as the reverse process of multiplication. This will enable you to understand more clearly the truths as indicated by these rules.

Solve on pages 54 and 55 problems 5, 8, 16, 23, 26, 27, 30, 33, 36, and 38.

ASSIGNMENT 13

Study carefully the rules (1) and (2) under §36 on page 55, also the **note** underneath these rules. Your attention is especially called to the word **transposition.** You should thoroughly understand the meaning of this word as used in algebra study.

Under §37 solve exercises 1, 5, 11, 16, and 18. Under §38 solve exercises 1 and 3.

CHAPTER IV

ADDITION AND SUBTRACTION OF LITERAL EXPRESSIONS

ASSIGNMENT 14

Study the rule given under §39. Understand what is meant by the **coefficient** of a term. This has been previously explained. Remember that only **similar** terms can be added and subtracted.

Solve the following exercises beginning on page 60: 4, 8, 10, the first and last of 15, 18, 21, 27, 32, 35, 41, and 47.

ASSIGNMENT 15

Study the rule given under §40 and solve the following problems on pages 63 and 64: 4, 6, 9, 12, 13, and the first two parts of 15.

ASSIGNMENT 16

It is convenient at times—especially in an examination—to be able to determine whether answers are right or wrong. A method for checking the addition of polynomials is given under §41. After a thorough study of this method solve the following exercises beginning on page 65 and **check the results**: 3, 7, 13, 15, 19, 23, 25, and 26.

ASSIGNMENT 17

Study the rule given under §42 and solve, beginning on page 66, exercises 1, 5, 8, 11, and 12.

Study the rule given under §43, also the two problems which are worked out; and then solve the following problems on page 68: 2, 4, 8, 18, 22, and 24.

ASSIGNMENT 18

Solve exercises 29, 30, 36, 41, 44, 46, and 48 on page 69.

ASSIGNMENT 19

Preceding the exercises on page 70 you will find three rules showing what should be done when grouping signs are to be removed. These rules must be carefully studied and thoroughly understood. Your attention is especially called to the method to be followed when signs of grouping are inclosed within other signs of grouping. Begin with the innermost sign and work outwardly.

Solve the following exercises under §44: 2, 12, 24, 26 and 33.

Study the rule under §45 and do exercises 4, 6, and 10.

CHAPTER V

MULTIPLICATION AND DIVISION OF LITERAL EXPRESSIONS

ASSIGNMENT 20

All rules and other information given under sections 46, 47, 48, and 49 must be carefully studied and thoroughly understood. Don't merely read these rules. Study them very thoroughly.

Solve the following exercises in §49: 2, 7, 18, 25, 36, 40, 42, 44, and 49.

ASSIGNMENT 21

Above the exercises on page 77 a rule is given and an example is worked out. You are to study these carefully and then

apply the principles to the following exercises on pages 77 and 78: 1, 5, 14, 21, 26, 35, 42, 44, 50, and 57.

If there is opportunity study exercises 58, 59, 60 and 61. The principle presented here is interesting to the mathematical student.

ASSIGNMENT 22

The rules and explanations under §51 deserve your close study if you are to apply properly the principles involved. As soon as you have thoroughly mastered the methods presented here apply them to the solution of the following exercises beginning on page 81: 2, 10, 24, 29, 34, 35, 40, 41, 47, and 50.

ASSIGNMENT 23

Solve the following exercises on pages 83 and 84: 52, 55, 59, 63, 65, 68, 70, 71, and 76.

ASSIGNMENT 24

Study carefully the rule given under §52. Note the connection between the process of cancellation and the rule for subtracting exponents in division. If you don't understand the process of cancellation find out about it from your instructor or your arithmetic.

The rule given under §53 is important. Study it carefully. Solve the following exercises: 1, 6, 12, 17, 20, 21, 28, 30, 34, 37, and 40.

ASSIGNMENT 25

A rule and a solved example precede the exercises on page 86. Study these very carefully and then solve the following exercises: 1, 5, 6, 12, 14, 16, 18, 21, 24, 28, and 30.

ASSIGNMENT 26

The method to be employed when one polynomial is to be divided by another is very fully explained on pages 87 and 88 under §55. This explanation should be studied in conjunction with the four rules given on page 88 and the application of these rules to the problems which are worked out, understood. As soon as the application of these rules to an actual solution is understood proceed to solve the following exercises: 1, 4, 12, 19, 22, 29, 32, 42, 44, and 47.

ASSIGNMENTS 27 AND 28

These are review assignments. It is expected that all of the Supplementary Exercises on pages 90 and 91 (or nearly all if time is limited) shall be solved. And not only must they be solved but the rules and principles involved must be thoroughly understood.

Suggestion for the teacher: It is suggested that after these review exercises have been completed a test be given covering the processes of addition, subtraction, multiplication, and division.

CHAPTER VI

LINEAR EQUATIONS: PROBLEMS

ASSIGNMENT 29

Study §57 very carefully and before you leave it be able to define: the **degree of a term**, the **degree of an equation**, a **linear equation**, a **quadratic equation**, and a **cubic equation**.

Study the examples that are worked out on page 93 and then solve the following exercises on that page and the following page: 1, 5, 8, 15, 18, 21, 24, and 25.

ASSIGNMENT 30

The solution of applied problems requires a considerable amount of thought and care on the part of the student. Make every effort to do these exercises. They are in a way a test of your ability to apply algebraic principles to practical work.

Solve the following exercises beginning on page 95: 1, 5, 7, 10, 12, 15, 18, and 20.

ASSIGNMENT 31

Solve the following exercises on pages 97 and 98: 23, 25, 27, 30, 32, 34, 36, and 37.

ASSIGNMENT 32

Solve the following problems under §60, beginning on page 98: 2, 5, 7, 11, 13, and 15.

ASSIGNMENT 33

Solve the following problems under §61, beginning on page 100: 1, 3, 6, 9, 13, 16, 18, and 22.

ASSIGNMENT 34

Solve the following problems under §62. Before proceeding with the solutions study carefully the two rules given above the exercises on page 104. The exercises to be solved are: 1, 3, 6, 10, and 12.

ASSIGNMENT 35

Solve exercises 1, 3, 6, 7, 9, and 11 under §63.

ASSIGNMENT 36

Digit problems generally require a little thought at first. Understand how a number is formed from its digits as explained at the bottom of page 107 and you should have little difficulty solving digit problems. Exercises 2, 3, 5, 7, and 9 on page 108 are to be solved.

ASSIGNMENT 37

Study thoroughly the definitions and explanations given on pages 109 and 110. Before leaving these for the solution of problems, be able to define **simultaneous equations, systems of simultaneous equations,** and **elimination.** Two rules are given, both of which require considerable study on your part.

Solve exercises 1, 4, 5, 8, 10, and 12.

ASSIGNMENT 38

Apply the principles used in assignment 37 to the applied problems of this assignment. Solve exercises 2, 3, 5, 7, 9, 10, and 15 under §69.

NOTE: It is suggested that two or three recitations at this point be devoted to the supplementary exercises on pages 114, 115, and 116.

CHAPTER VII

SPECIAL PRODUCTS AND QUOTIENTS

ASSIGNMENT 39

The rules given in §71 must be learned if the problems of the exercise following are to be satisfactorily solved. As soon as they have been learned, start with exercise 1 on page 118 and solve every other exercise.

ASSIGNMENT 40

Study all of the definitions and explanations given under §72. The rules should be learned and you should be able to define and explain the meaning of the terms: **square root, cube root, radical sign,** and **index.** Your attention is especially called to the statement bearing on the **double sign** $\pm$. The square root of 16 is either $+4$ or -4 because either of these multiplied together will give 16.

Beginning with 1, do every other exercise under §72.

ASSIGNMENT 41

Study the solved examples under §73, making note of the fact that a $\pm$ value is obtained. With this important principle in mind, solve exercises 1, 5, 8, 10, 11, 13, 16, and 18.

ASSIGNMENT 42

Learn the rule given at the top of page 123 and apply it to the following exercises on page 123: 1, 7, 8, 15, 17, 20, 26, 31, and 34.

Study the following solution of exercise 35.

35. $(a^2 - b^2) \div (a + b) = ?$

Solution: This may be written $\dfrac{a^2 - b^2}{a + b}$

$$\frac{a^2 - b^2}{a + b} = \frac{(a - b)\ (a + b)}{(a + b)} = a - b.$$

$a + b$ cancels from both the numerator and the denominator of the fraction.

Solve the following exercises on page 124: 39, 41, 44, 45, and 47.

Under §75 solve exercises 1, 9, 18, 20, and 24.

ASSIGNMENT 43

Learn the rule under §76 and study its application as illustrated by examples 1 and 2. After this is thoroughly understood, solve the following exercises on pages 125 and 126: 4, 7, 11, 21, 26, 32, 43, 57, 60, 62, 64, 70, 73, and 75.

ASSIGNMENT 44

It is possible to obtain the products of many arithmetical numbers by the rules of the preceding assignment. Study the rule given at the top of page 127 and apply it to the following exercises: 1, 4, 9, 14, 17, and 19.

Study carefully the rule given at the top of page 128 and understand its application to a problem as illustrated by the problem which is solved. After this is understood, solve the following exercises under §78: 2, 7, 10, 16, 22, 28, 36, 38, 41, 42, and 45.

ASSIGNMENT 45

Study carefully the rule for the square of a polynomial under §80. After understanding its application as illustrated by the solved example at the bottom of the page, solve the following exercises at the top of page 130: 1, 2, 4, 6, 9, 11, and 12.

ASSIGNMENT 46

A rule and an explanatory example are given under §81 which you should study very carefully. As you study the explanatory example note that $2a$ is the cube root of the first term ($8a^3$) and $3b$ is the cube root of the second term ($27b^3$).

Solve the following exercises under this paragraph: 1, 4, 5, 9, 11, 16, 21, 22, 27, 30, 31, and 34.

ASSIGNMENT 47

A study of the rule and explanatory example at the bottom of page 131 is required here. Establish in mind the difference between this rule and the one used in the preceding assignment. Note that the difference as far as the result is concerned is that the sign of the middle term is — instead of + as in the other case.

Solve the following exercises under §82: 1, 4, 5, 8, 11, 16, 26, 28, 31, and 35.

ASSIGNMENT 48

The supplementary exercises on pages 133, 134 and 135 furnish an excellent opportunity for the application of the rules given in Chapter VII. Before attempting the solution of the exercises called for, review the rules and then try to solve the exercises without referring to the rules. Consider this in the nature of a test of your ability to solve problems of this type.

Solve exercises 1, 3, 5, 7, 9, 16, 17, 20, 24, 31, 38, 42, and 49.

CHAPTER VIII

FACTORS. MULTIPLES. EQUATIONS SOLVED BY FACTORING

The subject of factoring is one of the most important in algebra and it is highly important that the meaning of the term factor be understood before proceeding with the solution of problems.

To factor an algebraic expression is to find two or more expressions which, when multiplied together, will produce the given expression.

Thus $20x^3y^2z$ may be written $2 \cdot 2 \cdot 5 \cdot x \cdot x \cdot x \cdot y \cdot y \cdot z$.

$20x^3y^2z$ is a monomial and $2, 2, 5, x, x, x, y, y, z$ are the factors of it.

The factors of 6 are 2 and 3; of 10, 2 and 5; of 12, 2, 2, 3; of mn, m and n; of $v\ (v+t)$, v and $v+t$.

A common factor of two or more expressions is an expression which is contained an exact number of times, without remainder, in each of them.

Thus in $4a^2b$, $8\ (m+n)$ and $16\ (x^2+2xy+y^2)$, 4 is a common factor. Be sure that you understand that this is so.

ASSIGNMENT 49

Study carefully all the rules, definitions and explanations given under §83 and §84 and apply them to the following problems on pages 137 and 138: 2, 5, 7, 13, 21, 24, 30, 34, the first three of 35, and the first and last two of 37.

ASSIGNMENT 50

Study the two explanatory examples under §85 and the rule and explanatory matter at the top of page 139.

Solve the following exercises on pages 139 and 140: 1, 2, 4, 6, 9, 14, 17, 21, 23, 25, and 30.

ASSIGNMENT 51

Study carefully all of the rules given under §86 and understand their application to the examples which are worked out. Apply the principles learned to the following exercises on page 141: 1, 3, 7, 11, 16, 19, 22, 25, 26, 28, and 30.

ASSIGNMENT 52

Solve the following problems on page 142: 31, 35, 36, 41, 44, 46, 51, 53, 56, 59, and 61.

ASSIGNMENT 53

Study carefully the rule given under §87 and be sure that you understand its application to the problems which are worked out. Solve the following exercises on pages 143 and 144: 5, 9, 15, 25, 31, 37, 42, 46, 50, 56, 58, 66, and 74.

Study the following solution of exercise 64,

$$\begin{aligned} ab^2 - 3ab - 70a &= a\,(b^2 - 3b - 70) \\ &= a\,(b - 10)\,(b + 7) \end{aligned}$$

ASSIGNMENT 54

The rules and explanations under §88 require concentrated study on your part. Note that the principal difference between the problems of this assignment and those of the previous one is that the first term here has a numerical coefficient greater than 1, while the first term of the others does not. The examples which are solved should illustrate the application of the rules given. Solve the following exercises in this section: 1, 5, 11, 17, 26, 31, 32, 38, 44, 45, 49, and 50.

ASSIGNMENT 55

Study the rule given under §89, understand its application to the explanatory problem and solve the following exercises: 2, 4, 6, 12, 19, 22, 29, 32, 34, 39, 40, 42, and 44.

Study the following solution of exercise 27,

$$\begin{aligned} 50a^2 - 32b^2 &= 2\,(25a^2 - 16b^2) \\ &= 2\,(5a - 4b)\,(5a + 4b). \end{aligned}$$

ASSIGNMENT 56

The exercises of this assignment are to be solved using the rule of the preceding assignment. Study the rules and illustrated examples very carefully.

You must learn to arrange the problem in the form of the difference of two squares if it is not already so arranged and you must remember that if the last term is in parentheses, since it is preceded by a minus sign, the signs of the terms will change when the parentheses are removed. Study the solved example No. 2 on page 149 to obtain an understanding of this. Solve exercises 1, 2, 5, 6, 9, 11, 15, 17, 19, and 21.

ASSIGNMENT 57

Solve exercises 25, 27, 29, 31, 33, and 36 under §90.

ASSIGNMENT 58

Study the rule and explanatory problem under §91. Study in connection with the exercises of this assignment those under §81 on pages 130 and 131. Solve the following problems on page 151: 2, 3, 6, 9, 12, 14, 16, 20, 22, and 25.

ASSIGNMENT 59

Study the rule and explanatory problem under §92 and in connection with the exercises of this assignment those under §82 on page 132. Solve the following problems on page 152: 1, 3, 7, 10, 15, 17, 20, 23, 27, and 30.

ASSIGNMENT 60

Some expressions which at first glance seem to be of a type which cannot be factored are nevertheless factorable. Take for example the quantity $y^4 + 64$.

Both of the terms of this expression are perfect squares. If there was a middle term, $16y^2$, the expression itself would

be a perfect square. Let us therefore, make the expression a perfect square remembering as we do so that if we add anything to a quantity we must subtract it again so that the value of the quantity will not be changed.

Hence, $y^4 + 64$ may be written,

$$y^4 + 16y^2 + 64 - 16y^2.$$

This is in the form of the difference of two perfect squares and may be written, $(y^2 + 8)^2 - 16y^2$.

$$(y^2 + 8)^2 - 16y^2 = (y^2 + 8 - 4y)\ (y^2 + 8 + 4y).$$

An expression such as $x^4 - 23x^2 + 1$ would be treated in the same way.

$x^4 - 23x^2 + 1$ would be a perfect square if the middle term was $2x^2$. To make $-23x^2$, $2x^2$, it is necessary to add $+25x^2$ to it. In order not to change the value of the expression $25x^2$ must be subtracted again. We then have,

$$\begin{aligned} x^4 - 23x^2 + 1 &= x^4 + 2x^2 + 1 - 25x^2 \\ &= (x^2 + 1)^2 - 25x^2 = (x^2 + 1 - 5x)\,(x^2 + 1 + 5x). \end{aligned}$$

Solve the following problems:

1. $x^4 - 7x^2 + 1.$
2. $36a^4 - 61a^2b^2 + 25b^4.$
3. $16x^4 + 4y^4z^4.$
4. $25a^4b^4 - 51a^2b^2 + 25.$

ASSIGNMENT 61

Many expressions may be factored by employing the so-called Theory of Divisor Method. The best way to illustrate this method is to solve a problem.

Suppose it be required to factor $6a^5 + 19a^4 - 20a^3 - 65a^2 + 24a + 36$.

The first thing to do is to separate the term containing no unknown values (in this case 36) into its exact divisors. The exact divisors of $+36$ are $+36, +1; -36, -1; -12, -3; +12, +3; -2, -18; +2, +18$, etc.

These divisors must be substituted in the original expression as values of a until one is found which will make the expression disappear.

If $a = +1$, the expression will vanish as indicated below,

$$6(1)^5 + 19(1)^4 - 20(1)^3 - 65(1)^2 + 24(1) + 36$$
$$= 6 + 19 - 20 - 65 + 24 + 36 = 0.$$

One of the factors of the expression therefore is $a - 1$. If the original expression is divided by $a - 1$, the quotient obtained will be equivalent to the product of the remaining factors. This quotient must again be factored, if possible, and all factors obtained be again separated into factors until no more factors can be obtained.

Study the following solution to obtain an understanding of this process. Remember that the first factor was $a - 1$.

We now divide the original expression by $a - 1$.

$$
\begin{array}{l}
\underline{a-1)}\ 6a^5+19a^4-20a^3-65a^2+24a+36\ \underline{(6a^4+25a^3+5a^2-60a-36} \\
\qquad \underline{6a^5-6a^4} \\
\qquad\qquad 25a^4-20a^3 \\
\qquad\qquad \underline{25a^4-25a^3} \\
\qquad\qquad\qquad 5a^3-65a^2 \\
\qquad\qquad\qquad \underline{5a^3-5a^2} \\
\qquad\qquad\qquad\quad -60a^2+24a \\
\qquad\qquad\qquad\quad \underline{-60a^2+60a} \\
\qquad\qquad\qquad\qquad\quad -36a+36 \\
\qquad\qquad\qquad\qquad\quad \underline{-36a+36}
\end{array}
$$

The quotient, $6a^4 + 25a^3 + 5a^2 - 60a - 36$ must be treated in the same way as the original equation. The divisor of -36 which will make this equation = zero is -2. Therefore, the factor to be used as a divisor is $a + 2$. We then have,

$$
\begin{array}{rl}
\underline{a+2)} & 6a^4 + 25a^3 + 5a^2 - 60a - 36 \ \underline{)6a^3 - 13a^2 - 21a - 18} \\
& \underline{6a^4 + 12a^3} \\
& \qquad 13a^3 + 5a^2 \\
& \qquad \underline{13a^3 + 26a^2} \\
& \qquad\qquad -21a^2 - 60a \\
& \qquad\qquad \underline{-21a^2 - 42a} \\
& \qquad\qquad\qquad -18a - 36 \\
& \qquad\qquad\qquad \underline{-18a - 36}
\end{array}
$$

We have now obtained three factors of the original expression, namely, $a - 1$, $a + 2$ and $6a^3 + 13a^2 - 21a - 18$. The last factor can be further factored by the same method. The divisor of -18 which will make the expression disappear is -3. The factor to be used as a divisor is therefore $a + 3$. We then have,

$$
\begin{array}{rl}
\underline{a+3)} & 6a^3 + 13a^2 - 21a - 18 \ \underline{(6a^2 - 5a - 6} \\
& \underline{6a^3 + 18a^2} \\
& \qquad -5a^2 - 21a \\
& \qquad \underline{-5a^2 - 15a} \\
& \qquad\qquad -6a - 18 \\
& \qquad\qquad \underline{-6a - 18}
\end{array}
$$

$6a^2 - 5a - 6$ may be factored using the method of assignment 53. $(6)(-6) = -36$. Are there two factors of -36 which added together will give -5? $+4$ and -9 will give this result.

$$
\begin{aligned}
\text{Then } 6a^2 - 5a - 6 &= 6a^2 - 9a + 4a - 6 \\
&= (6a^2 - 9a) + (4a - 6) \\
&= 3a(2a - 3) + 2(2a - 3) \\
&= (3a + 2)(2a - 3).
\end{aligned}
$$

Therefore the factors of $6a^5 + 19a^4 - 20a^3 - 65a^2 + 24a + 36$ are

$$a - 1,\ a + 2,\ a + 3,\ 3a + 2,\ 2a - 3.$$

Factor the following problems:

1. $x^3 - 9x^2 + 26x - 24$.
2. $4m^5 - 22m^4 + 17m^3 + 83m^2 - 152m + 160$.

After you thoroughly understand the principle, you can do the factoring by testing the original expression for factors until you have as many different factors as the number of the degree of the expression. This will save dividing. If you can not find this number of factors, however, this indicates that some factor must be an expression of the second degree, and you will have to resort to the dividing to discover this factor.

ASSIGNMENT 62

Study carefully the General Suggestions on Factoring, §93 on pages 152 and 153, and beginning with exercise 1 on page 153 do at least every other problem.

ASSIGNMENT 63

Beginning with exercise 31 on page 154 solve every other problem, including number 65.

ASSIGNMENT 64

Beginning with exercise 67 on page 154 solve every other exercise, including number 99.

HIGHEST COMMON FACTOR AND LOWEST COMMON MULTIPLE

ASSIGNMENT 65

Understand from the beginning what is meant by a **Common Factor** of two or more expressions and what is meant by the **Highest Common Factor.** Be absolutely sure that you

understand that a common factor would not necessarily be the highest common factor.

Learn the rules at the bottom of page 155, study the explanatory example, and solve the following exercises on page 156: 2, 3, 5, 9, 10, 12, 13, 16, and 18.

ASSIGNMENT 66

Solve exercises 19, 21, 23, 24, and 25 on page 156.

ASSIGNMENT 67

Study carefully the definition of **Lowest Common Multiple** in §95 at the bottom of page 156. Examples 1 and 2 at the top of page 157 show how the L. C. M. is to be found. Establish in your mind the connection between lowest common denominator as you studied it when solving fractions in arithmetic and lowest common multiple as you are now studying it.

Learn the rule preceding the exercises on page 157, and solve the following exercises: 2, 4, 7, 9, 11, 13, 15, and 17.

ASSIGNMENT 68

Solve exercises 19, 22, 23, 25, 27, 29, and 30.

ASSIGNMENT 69

When expressions can not readily be factored, the method of Continued Long Division, illustrated below, may be used.

To find the H. C. F. (or L. C. M.) of three or more such quantities, find the H. C. F. or (L. C. M.) of two of them; then of this result and the third quantity; and so on. The last H. C. F. or (L. C. M.) thus obtained is the one required.

The process is best shown by the actual solution of a problem. Find the *H. C. F.* and *L. C. M.* of $4a^3 + a - 1$ and $6a^3 + a^2 - 1$.

$$
\begin{array}{r@{\,}l}
 & 6a^3 + a^2 - 1 \\
 & \qquad\qquad\quad 2 \\
4a^3 + a - 1) & 12a^3 + 2a^2 - 2 \;(3 \\
 & 12a^3 + 3a - 3 \\
 & \qquad 2a^2 - 3a + 1
\end{array}
$$

Note that $6a^3 + a^2 - 1$ was multiplied by 2. This was necessary to obtain a coefficient in the first term of the dividend exactly divisible by the first term in the divisor.

We now divide the first divisor by the remainder,

$$
\begin{array}{r@{\,}l}
2a^2 - 3a + 1) & 4a^3 + a - 1 \;(2a + 3 \\
 & 4a^3 - 6a^2 + 2a \\
 & \qquad 6a^2 - a - 1 \\
 & \qquad 6a^2 - 9a + 3 \\
 & \qquad 4(+8a - 4 \\
 & \qquad\qquad 2a - 1
\end{array}
$$

Note that the remainder is divided by 4 to simplify it.

We now proceed as before. That is, divide the second divisor by the remainder

$$
\begin{array}{r@{\,}l}
2a - 1) & 2a^2 - 3a + 1 \;(a - 1 \\
 & 2a^2 - a \\
 & \quad -2a + 1 \\
 & \quad -2a + 1
\end{array}
$$

Therefore, the *H. C. F.* of the two original expressions is $2a - 1$.

To find the *L. C. M.* we factor each of these original expressions by the aid of the *H. C. F.*

$$
\begin{array}{r@{\,}l}
2a-1) & 4a^3+a-1 \;(2a^2+a+1 \\
 & 4a^3-2a^2 \\
 & \qquad 2a^2+a \\
 & \qquad 2a^2-a \\
 & \qquad\quad 2a-1 \\
 & \qquad\quad 2a-1
\end{array}
\qquad
\begin{array}{r@{\,}l}
2a-1) & 6a^3+a^2-1 \;(3a^2+2a+1 \\
 & 6a^3-3a^2 \\
 & \qquad 4a^2 \\
 & \qquad 4a^2-2a \\
 & \qquad\quad 2a-1 \\
 & \qquad\quad 2a-1
\end{array}
$$

The *L. C. M.* is $(2a - 1)(2a^2 + a + 1)(3a^2 + 2a + 1)$.

Solve the following problems.

Find *H. C. F.* and *L. C. M.* of:

1. $24y^2 - xy - 10x^2$, $15y^2 - xy - 6x^2$, $12y^2 + xy - 6x^2$.
2. $a^2 + 20a^4 - 1$, $75a^4 + 15a^3 - 3 - 3a$.
3. $6m^3 - m^2 + 1$, $2m^3 - 3m^2 + 1$, $4m^3 - 3m - 1$.

EQUATIONS SOLVED BY FACTORING

ASSIGNMENT 70

All of the rules and explanations under §96 are very important and you should devote a great deal of time to a study of them before attempting to solve the exercises.

Note particularly that before the equations are factored they are arranged so that the whole expression equals zero, and that the values of the unknown quantities are obtained from the factors.

As soon as you thoroughly understand the method of procedure, solve the following exercises on page 159: 3, 8, 9, 11, 15, and 20.

ASSIGNMENT 71

On the same page, solve exercises 21, 23, 26, 28, 31, 35, 37, and 40.

ASSIGNMENT 72

The applied problems on pages 160 and 161 are to be solved in the same way as the exercises on page 159, once the equations have been formed. Make every effort to form these equations without help. For this assignment solve the following exercises on page 160: 42, 43, 46, 49, and 51.

ASSIGNMENT 73

Solve exercises 52, 54, 55, and 58 on page 161.

ASSIGNMENT 74

Solve the following exercises on page 162: 3, 6, 8, 10, 15, 16, 18, and 20.

ASSIGNMENT 75

Solve the following exercises on page 162: 22, 24, 27, 28, 30, 33, and 35.

ASSIGNMENT 76

Solve the following exercises on page 163: 37, 39, 42, 43, 47, 50, 53, and the first, sixth, and last parts of 57.

ASSIGNMENT 77

No answers are given to the problems of this assignment. You may consider, as you solve them, that you are being tested as to your ability to solve factoring problems. Make a supreme effort to obtain correct solutions.

Factor:

1. $A^6 - B^6 - (A - B)^2$.
2. $m^4 + 64$.
3. $x^2 - a^2 - 2x - 4b^2 - 4ab + 1$.
4. $(x^3 + 8y^3)\ (x + y) - 6xy\ (x^2 - 2xy + 4y^2)$.
5. $R^4 - 23S^2 + S^4$.
6. $(m + n)^2 + (m + p)^2 - (p + q)^2 - (n + q)^2$.
7. Find the $H.\ C.\ F.$ and $L.\ C.\ M.$ of $(am - m^2)^3$ and $m^3 - a^2m$.
8. Find the $H.\ C.\ F.$ and $L.\ C.\ M.$ of $(4a^2 + 2a - 12)(a^3 + 1)$ and $2\ (4a^2 - 8a + 3)\ (a^2 + 2a)$.

CHAPTER IX

FRACTIONS

ASSIGNMENT 78

The words **Denominator, Numerator, Term,** etc., mean the same in algebra as in arithmetic. This is explained in §97.

It is doubtful, however, if the **Rules of Signs** are as thoroughly understood. These are explained in §98 and deserve your serious consideration. Do not proceed with the solution of fractional problems until you understand the application of these rules.

Solve the following exercises under §98: 2, 3, 7, 8, 9, 14, 17, 19, 23, 25, and 27.

ASSIGNMENT 79

The rule for reducing a fraction to its lowest terms is given at the bottom of page 166. Its application to several problems is also shown. Simply remember that all you have to do is to change both the numerator and denominator of the fraction into their respective factors and cancel the factors which are **common** to both.

Solve the following exercises on page 167: 4, 9, 11, 14, 18, 20, 21, 23, 29, and 32.

ASSIGNMENT 80

Solve exercises 34, 35, 38, 39, 41, and 42 on page 168.

ASSIGNMENT 81

Study carefully the definitions, explanations, and illustrative examples under §100. Understand thoroughly what the

terms **Integral Expression, Fractional Expression,** and **Mixed Expression** mean.

Learn the rule at the top of page 169 and solve the following exercises on the same page: 4, 7, 10, 16, 17, 21, 22, 24, and 25.

ASSIGNMENT 82

Study the rules, explanations and illustrations under §101, learn the rule on page 170 and apply it to the solution of exercises 2, 4, 7, 10, 12, and 13.

Study the Note just preceding the exercises.

ASSIGNMENT 83

Study all explanations and illustrations under §102. Understand that **equivalent** means "equal in value." You will find here an opportunity to apply your knowledge of lowest common multiple to the denominators of the fractions.

Solve the following exercises: 8, 10, 12, 14, 18, 20, and 21.

Study the following solution of exercise 7 to refresh your memory as to the methods which you previously used in arithmetic.

The problem is to reduce to equivalent fractions having the least common denominator, $\frac{1}{2}$, $\frac{3}{5}$, $\frac{7}{16}$, $\frac{5}{12}$.

The *L. C. M.* of 2, 5, 16, and 12 is 240.

$$\text{Then } \frac{1}{2} = \frac{120}{240},\ \frac{3}{5} = \frac{144}{240},\ \frac{7}{16} = \frac{105}{240},\ \text{and } \frac{5}{12} = \frac{100}{240}.$$

ASSIGNMENT 84

The **Addition** and **Subtraction** of **Fractions** is one of the most important processes considered during the progress of algebra study.

A mistake which commonly occurs and which you should

seek to avoid is that of forgetting to write down the common denominator after the sum or difference of the numerators has been found.

Study very carefully the examples which have been solved for your benefit on page 173, learn the rule at the bottom of the page and solve the following exercises on page 174: 3, 7, 9, 12, 15, 16, and 22.

ASSIGNMENT 85

Solve the following exercises on pages 174 and 175: 25, 27, 30, 31, and 34.

ASSIGNMENT 86

Solve exercises 39 and 40 on page 175 and exercises 42 and 45 on page 176.

ASSIGNMENT 87

You will remember that in arithmetic when you wished to change $3\frac{1}{2}$ (a mixed number) to an improper fraction, all you did was to multiply 3 by 2 and add the 1 to obtain the numerator of the improper fraction. You used the same denominator as in the fractional part of the mixed number and your result was $\frac{7}{2}$.

If you will study the solved example under §104 you will see that exactly the same process is used. Study this carefully.

Learn the rule at the bottom of page 176 and solve the following exercises: 1, 5, 9, 14, 15, and 16.

ASSIGNMENT 88

When you are called upon to multiply fractions use the process of **cancellation** as much as possible. This will save

you much extra work and time. If the example at the top of page 178 was written (after factoring),

$$\frac{(a+b)+(2a-b)}{(2a+b)+(a-b)} \times \frac{(a-b)+(3a+2b)}{(2a-b)+(a+3b)}$$

instead of as it is it would not be possible to cancel out the $(a-b)$ and $(2a-b)$. Why is this?

Learn the rule at the bottom of page 177 and solve the following exercises on page 178: 5, 12, 15, 21, 23, 25, and 26.

ASSIGNMENT 89

Solve the following exercises on page 179: 29, 32, 34, 35, and 38.

ASSIGNMENT 90

When mixed and fractional expressions are to be multiplied, (§106), change the mixed expressions to fractional form and proceed as in the previous assignment.

Study the example which is worked out above the exercises on page 180 and as soon as the method is understood solve the following problems: 1, 5, 8, 10, 13, and 14.

ASSIGNMENT 91

When you multiply a fraction by an integral expression about all you need to remember is that the numerator of the fraction is multiplied by the integer, the denominator remaining the same.

$$9 \times \frac{11}{2} \text{ is the same as } \frac{9 \times 11}{2} = \frac{99}{2}$$

The example which is worked shows how the integral ex-

pression may be considered a fraction with a denominator of 1 and then the rule for the multiplication of fractions applied. Study this and then solve the following exercises: 5, 7, 9, 11, and 14.

ASSIGNMENT 92

Solve exercises 16, 18, 20, 22, and 24 on page 182.

ASSIGNMENT 93

About the only thing you need to remember when you are required to divide one fraction by another is invert the divisor and multiply.

$\frac{7}{24} \div \frac{5}{6}$ is the same as $\frac{7}{24} \times \frac{6}{5} = \frac{7}{20}$.

Learn the rule at the bottom of page 182, study the illustrative example at the top of page 183, and solve the following exercises on page 183: 8, 9, 10, 13, 16, and 19.

ASSIGNMENT 94

Solve the following exercises on page 184: 22, 24, 27, 29, and 30.

ASSIGNMENT 95

The solution of complex fractions requires that he who attempts to solve them shall work with unusual care. The processes are the same as used when solving any fractions. About the only difference is that more of the same processes are used in one problem than in a simple fraction.

Study the following solution. As soon as you think you understand it, close the book and see if you can do it. Begin by simplifying the very last denominator.

Reduce to simplest form $2a-1-\cfrac{a-1}{2-\cfrac{a}{a-\cfrac{a}{1+a}}}$

$$2a-1-\cfrac{a-1}{2-\cfrac{a}{a-\cfrac{a}{1+a}}}=2a-1-\cfrac{a-1}{2-\cfrac{a}{\cfrac{a+a^2-a}{1+a}}}$$

$$=2a-1-\cfrac{a-1}{2-\cfrac{a\,(1+a)}{a^2}}$$

$$=2a-1-\cfrac{a-1}{\cfrac{2a^2-a-a^2}{a^2}}$$

$$=2a-1-\frac{a^2\,(a-1)}{a^2-a}$$

$$=2a-1-\frac{a^2\,(a-1)}{a\,(a-1)}$$

$$=2a-1-a$$

$$=a-1$$

If you can solve the above problem without help you should be able to do those exercises given on page 185. Study it thoroughly and then do exercises 3, 6, 7, 10, and 11 on page 185.

ASSIGNMENT 96

Solve exercises 14, 17, 20, and 21 on page 185. In addition, simplify to its lowest terms:

$$\left\{1+\cfrac{1+\cfrac{1+A}{1-3A}}{1-3\cfrac{1+A}{1-3A}}\right\}\div\left\{1-3\left(\cfrac{\cfrac{1+A}{1-3A}+1}{1-3\left(\cfrac{1+A}{1-3A}\right)}\right)\right\}$$

The answer is A.

NOTE: It is the opinion of the author of this manual that at this point several days should be devoted to the solution of general fractional problems. Excellent ones are given on pages 186, 187, and 188 and many more may be obtained from college entrance examination papers.

CHAPTER X

FRACTIONAL EQUATIONS. PROBLEMS. FORMULAE

ASSIGNMENT 97

Be sure that you understand what a fractional equation is and what is meant by the **clearing of fractions.** When you clear of fractions the **denominators of the fractions do not appear in the cleared equation.** Don't forget this.

There is a note at the bottom of page 190 which you must not only read but must remember.

Do the following exercises on page 190: 2, 4, 8, 15, 18, and 20.

ASSIGNMENT 98

Solve the following exercises on pages 191 and 192: 23, 27, 30, 35, 37, 38, and 42.

ASSIGNMENT 99

Before you can obtain the answer to a literal equation you know, of course, that from the problem as stated you must be able to determine what is to be found and also sufficient data to permit the formation of an equation.

You should now be able to solve fractional equations. What you need to do in order to cope successfully with the problems

of this type is to think. These solutions afford an excellent opportunity for you to test your ability to apply your knowledge of the principles of algebra to practical solutions.

Solve problems 2, 5, 7, 9, 11, and 13 under §112.

ASSIGNMENT 100

Solve problems 15, 17, 19, 21, 23, and 25 under §112.

ASSIGNMENT 101

Solve problems 27, 29, 31, 33, and 35 under §112.

ASSIGNMENT 102

Literal equations as discussed under §113 are solved in the same way as those from which a numerical answer was obtained. A study of the solved example on page 197 will show you that this is so.

Solve the following exercises under §113: 2, 4, 6, 8, and 11.

ASSIGNMENT 103

Solve the following exercises under §113: 13, 14, 17, 19, 23, 29, and 33.

ASSIGNMENT 104

Under "Supplementary Exercises" on page 202 solve exercises 1, 3, 5, 8, and 9.

ASSIGNMENT 105

On pages 203, 204, and 205, solve exercises 10, 13, 18, 22, 25, and 28.

NOTE: It is the author's opinion that several recitations should now be devoted to a General Review with perhaps a

test or two. The students should be required to solve several problems taken from college entrance examination papers. The student should be thrown on his own resources as much as possible.

CHAPTER XI

PROPORTION. VARIABLES

ASSIGNMENT 106

Study all definitions and explanations in sections 114 and 115. If you have studied as you should you should be able to define **Ratio, Antecedent, Consequent, Proportion, Terms of a Proportion, Means,** and **Extremes.** Close your book after you think you have studied sufficiently to master the meaning of these words, and see if you can define them.

Solve the following exercises in §114: 2, 5, 7, 12, 14, and 19.

Under §115 do exercises 3, 6, 8, 11, 13, and 15.

ASSIGNMENT 107

Study §116. Can you define and illustrate the meaning of **Mean Proportional, Third Proportional,** and **Fourth Proportional?**

Solve exercises 1, 5, 7, 9, 10, 17, and 21 on page 210.

Note: The remainder of this chapter should be covered by a short series of lectures by the instructor. It is not intended that the student should obtain a complete understanding of the subject of Ratio and Proportion in connection with algebra study because it will be much more fully discussed when the course in Plane Geometry is pursued at a subsequent period.

Two or three days devoted to lectures on the matter pre-

sented in this book with an accompanying assignment of a few problems to be done outside of class should give the student about as much of an understanding of Ratio and Proportion as he needs to have at this particular period of his progress in mathematics.

CHAPTER XII

SYSTEMS OF LINEAR EQUATIONS

(Sometimes Called Simultaneous Equations)

Understand what **Simultaneous Equations** are before attempting their solution. Note that they may be solved by several different methods. These will be discussed separately.

ASSIGNMENT 108

The unknown values of the problems of this assignment are to be solved by **Elimination** by either addition or subtraction. To illustrate the process we will solve exercise 3 on page 232.

$$\text{We have } \begin{cases} 2x - y = 5. & (1) \\ 5x - 2y = 14. & (2) \end{cases}$$

We must eliminate either x or y. We will eliminate y. To do this we can multiply (1) by 2 and then subtract (2) from (1). Doing this we have

$$\begin{array}{rr} & 4x - 2y = 10. \\ & \underline{5x - 2y = 14.} \\ \text{Subtracting,} & -x = -4. \\ & \text{or } x = 4. \end{array}$$

To obtain the value of y we substitute the value of x in either of the original equations. Suppose we use (1).

$$\begin{aligned} \text{Then } 16 - 2y &= 10. \\ -2y &= -16 + 10. \\ -2y &= -6. \\ y &= 3. \end{aligned}$$

Are these answers correct? To determine whether they are or not, we may substitute the found values of x and y (4 and 3) in the original equations. If they satisfy the equations they are correct.

Substituting in (1) we have $8 - 3 = 5$.
Substituting in (2) we have $20 - 6 = 14$.
This proves that the values are correct.

Solve exercises 1, 5, 7, and 9 under §126.

ASSIGNMENT 109

Solve exercises 11, 13, 15, 17, and 19 under §126.
When you do number 19, clear of fractions before you solve.

ASSIGNMENT 110

Study the method of elimination by Comparison as shown by the solved example at the bottom of page 233. Study the rule given at the bottom of the page and apply it to the solution of exercises 1, 3, 5, 7, and 9 on page 234.

ASSIGNMENT 111

Solve exercises 11, 13, 17, 21, and 23 on page 234.

ASSIGNMENT 112

Study the method of elimination by Substitution as explained under §128. The rule just preceding the exercises explains how this method is to be applied to the solution of a problem. Study this very carefully.

Solve exercises 1, 3, 5, 7, and 9 on page 235 using this method.

ASSIGNMENT 113

Solve exercises 12, 13, 15, 21, and 23 on page 236 using the Substitution method.

ASSIGNMENT 114

Solve exercises 1, 3, 6, 8, 9, and 11 under "Miscellaneous Exercises" on page 236. Be sure to use all three methods during the progress of these solutions.

ASSIGNMENT 115

Solve problems 2, 4, 6, 8, and 10 on page 237.

ASSIGNMENT 116

Solve problems 12, 14, 16, 20, 24, and 26 on pages 238 and 239.

ASSIGNMENT 117

Solve problems 30, 32, 36, 39, and 43 on pages 240 and 241.

ASSIGNMENT 118

The results of the exercises under §130 are literal instead of numerical. The usual methods of elimination may be used

here just as in problems from which a numerical answer is obtained but as a general thing the addition or subtraction method is employed.

Study carefully the example solved above the exercises. As soon as you understand the method do the following exercises on page 242: 1, 3, 5, 7, 9, and 11.

ASSIGNMENT 119

Solve exercises 14, 16, 18, 20, and 22 on page 243.

ASSIGNMENT 120

Study §131 to obtain an understanding of the terms, **Inconsistent** and **Equivalent** equations.

Also study §132 to find out how you may solve simultaneous equations graphically. Note that the graph of each equation is a straight line. The point of intersection of the lines determines the value of the unknown quantities. Study the explanation which accompanies the figure at the top of page 245.

Supply yourself with a sheet of graph paper and solve graphically problems 1, 3, 6, and 8.

ASSIGNMENT 121

Take a piece of graph paper and solve exercises 14, 16, 18, and 20 on page 246.

ASSIGNMENT 122

Note carefully what is printed in italics under §133. Study the examples which are solved and solve exercises 1, 3, 5, and 8 in the same manner.

ASSIGNMENT 123

On page 248 solve exercises 11, 15, 17, and 19.

ASSIGNMENT 124

When three unknown quantities are present there are three equations. (See §134.) These three equations must be reduced to two and the two equations to one before any unknown value can be found. A study of the solved example will show you how this is accomplished.

Solve exercises 1, 3, 6, and 8 on page 250.

ASSIGNMENT 125

Study very carefully the following solution of exercise 9 on page 250. As soon as you understand it proceed to solve exercise 11 in the same manner. In addition to this solve exercises 10, 13, 17, and 20.

$$9. \quad \begin{cases} \frac{a}{5} + \frac{b}{2} + \frac{c}{3} = 19. \\ \frac{a}{10} - \frac{b}{10} + \frac{c}{6} = 6. \\ \frac{a}{4} + \frac{b}{5} - \frac{c}{15} = 5. \end{cases}$$

$$\text{Clearing fractions we have} \begin{cases} 6a + 15b + 10c = 570. & (1) \\ 6a - 6b + 10c = 360. & (2) \\ 15a + 12b - 4c = 300. & (3) \end{cases}$$

Subtracting (2) from (1), we have

$$\begin{aligned} 6a + 15b + 10c &= 570. \\ 6a - 6b + 10c &= 360. \\ \hline 21b &= 210. \\ b &= 10. \end{aligned}$$

To eliminate a from (2) and (3) we multiply (2) by 10 and (3) by 4 and then subtract.

$$
\begin{array}{lrl}
\text{That gives us} & 60a - 60b + 100c &= 3600. \\
& \underline{60a + 48b - 16c} &\underline{= 1200.} \\
\text{Subtracting,} & -108b + 116c &= 2400. \quad (4)
\end{array}
$$

Substituting the value of b in (4), $-1080 + 116c = 2400.$

$$116c = 3480.$$

$$c = 30.$$

To get the value of a, the values of b and c may be substituted in either (1), (2), or (3).

Suppose we take (1). Then $6a + 150 + 300 = 570.$

$$6a = 570 - 450.$$

$$6a = 120.$$

$$a = 20.$$

NOTE: When you solve exercise 10 do not clear of fractions.

ASSIGNMENT 126

Solve exercises 1, 3, 5, and 8 under "Supplementary Exercises" on page 253.

ASSIGNMENT 127

Note that on page 253 exercises 9, 10, 11, and 12 contain four unknowns. Note also that each system has four equations. It is evident that these must be reduced to one equation before a solution can be obtained. Study the illustration and suggestions given below, after which solve problems 9, 11, and 12 on page 253.

Study this solution and apply the method to problem 9.

$$
\text{Solve} \quad \begin{cases}
x + y + u = 6. & (1) \\
x + y + z = 7. & (2) \\
u + z + y = 8. & (3) \\
x + u + z = 9. & (4)
\end{cases}
$$

Adding, we have $3x + 3u + 3y + 3z = 30.$

Or, $x + u + y + z = 10.$ (5)

Subtracting (2) from (5), $u = 3.$
Subtracting (3) from (5), $x = 2.$
Subtracting (4) from (5), $y = 1.$
Subtracting (1) from (5), $z = 4.$

To solve a problem like 11 or 12 on page 253, you must reduce the four equations to three equations, containing three unknowns; the three equations to two equations, containing two unknowns; and the two equations to one equation, containing one unknown.

For example, in problem 12, eliminate p from (1) and (2), then from (2) and (3), then from (3) and (4). Then eliminate q from the three equations obtained, r from the two resulting equations and obtain a value for s.

After one value is obtained, simply substitute back in the equations to obtain the other values.

Understand that you do not necessarily have to eliminate p first. Another letter may be chosen. This is simply a suggestion.

ASSIGNMENT 128

Under "Supplementary Exercises" on page 253 solve problems 13, 15, and 17.

CHAPTER XIII

SQUARE ROOT. QUADRATIC SURDS

ASSIGNMENT 129

Study carefully the explanations, solved examples, and rules under §135. The five rules at the top of page 257 deserve

your special consideration. As soon as you are sure that you thoroughly understand these, solve the following problems on page 257: 1, 4, 7, and 10.

ASSIGNMENT 130

Solve problems 14, 16, and 18 on page 258.

ASSIGNMENT 131

Study the following solution to find out how the square root of a quantity involving fractions is obtained.

Find the square root of:

$$\frac{A^4}{16}+\frac{A^3}{4B}+\frac{3A^2}{20B^2}-\frac{A}{5B^3}+\frac{1}{25B^4}$$

$$\begin{array}{r|l|l}
 & \frac{A^4}{16}+\frac{A^3}{4B}+\frac{3A^2}{20B^2}-\frac{A}{5B^3}+\frac{1}{25B^4} & \frac{A^2}{4}+\frac{A}{2B}-\frac{1}{5B^2} \\
 & \frac{A^4}{16} & \\
2\left(\frac{A^2}{4}\right)+\frac{A^3}{4B}\div\frac{A^2}{2}=\frac{A^2}{2}+\frac{A}{2B} & \frac{A^3}{4B}+\frac{3A^2}{20B^2} & \\
 & \frac{A^3}{4B}+\frac{A^2}{4B^2} & \\
2\left(\frac{A^2}{4}+\frac{A}{2B}\right)+\left(-\frac{A^2}{10B^2}\div\frac{A^2}{2}\right)=\frac{A^2}{2}+\frac{A}{B}-\frac{1}{5B^2} & -\frac{A^2}{10B^2}-\frac{A}{5B^3}+\frac{1}{25B^4} & \\
 & -\frac{A^2}{10B^2}-\frac{A}{5B^3}+\frac{1}{25B^4} &
\end{array}$$

Find the square roots of the following expressions:

1. $\frac{25}{4}-\frac{15x}{y}+\frac{41x^2}{4y^2}-\frac{3x^3}{2y^3}+\frac{x^4}{16y^4}$

Ans. $\frac{5}{2}-\frac{3x}{y}+\frac{x^2}{4y^2}$

2. $\frac{M^2}{N^2}+\frac{2M}{N}+3+\frac{2N}{M}+\frac{N^2}{M^2}$

3. $\frac{4a^5}{9}-2a^3-\frac{17a^4}{9}+\frac{9a}{4}+\frac{4a^6}{9}+\frac{9}{16}+\frac{7a^2}{4}$

ASSIGNMENT 132

Study carefully all of the explanations, rules, and illustrative solutions under §136, and solve exercises 1, 8, 12, 15, and 20 on page 260.

ASSIGNMENT 133

Solve the following problems.

Find the square root of:

1. 44994.8944. Ans. 212.12.
2. .0000879844. Ans. .00938.

Find to three decimal places the square root of:

3. $\frac{2}{3}$. Ans. .816 +.
4. $\sqrt{\sqrt{5}+6}$. Ans. 2.869 +.

ASSIGNMENT 134

Learn the definitions on page 261 and all of the explanations, solved examples, and rules on page 262. As soon as you understand how a surd can be reduced to its simplest form, proceed with the solution of the following exercises under §139: 8, 15, 21, 30, 37, 40, and 42.

ASSIGNMENT 135

Learn the rule given under §140 and apply it to exercises 6, 10, 13, 20, 24, and 29.

ASSIGNMENT 136

Learn the rule under §141 and apply it to exercises 8, 12, 15, 24, 30, 33, and 36.

ASSIGNMENT 137

Study the explanations and illustrative problems under §142 and apply the methods indicated to the solution of exercises 2, 5, 11, 15, and 21 on page 266.

ASSIGNMENT 138

Solve problems 23, 25, 27, 29, and 30 on page 266.

NOTE: The suggestion is made at this point that a day or two be devoted to an explanation by the instructor of the use of Imaginary and Complex numbers. These subjects are thoroughly covered during the second year of algebra study. Nevertheless, because of their relation to the general subject now under discussion it is not out of place to give some attention to them here.

CHAPTERS XIV and XV

QUADRATIC EQUATIONS

The instructor should so plan his work that he can devote some little time to a discussion of the rules and their application as indicated by the material contained in these two chapters.

The opinion of the author of this manual is that the subject of quadratic equations is still considered a second year subject as far as a complete discussion of it is concerned; although

the present opinion seems to be in favor of devoting some time to a discussion of it as the first year's work draws to a close. Possibly the reason for this is that it is desired to form a sort of connecting link between first and second year algebra.

CHAPTER XVI

EXPONENTS

ASSIGNMENT 139

Understand first of all what an **Exponent** is. You should know that when like factors are multiplied together, the exponent of the result is obtained by adding the exponents of the factors multiplied.

$$t^{20} \times t^{4} = t^{24}.$$
$$k^{12} \times k^{12} \times k^{12} = k^{36}.$$
$$a^{n} \times a^{n-1} \times a^{n+2} = a^{(n+n-1+n-2)} = a^{3n-3}.$$

Solve exercises 2, 5, 11, 14, 17, 19, and 21 under §156.

ASSIGNMENT 140

When like factors are divided the exponent of the quotient is obtained by subtracting the exponent of the divisor from that of the dividend.

$$\frac{A^{12}}{A^{5}} = A^{12-5} = A^{7}.$$
$$\frac{W^{5n}}{W^{n}} = W^{5n-n} = W^{4n}.$$
$$\frac{X^{m+1}}{X^{2m-1}} = X^{(m+1)-(2m-1)} = X^{m+1-2m+1} = X^{2-m}.$$

Solve exercises 3, 6, 9, 14, 15, 17, 19, and 21 under §157.

ASSIGNMENT 141

Study very carefully indeed the rule and solved problem under §158. As soon as you are sure that you understand the process solve exercises 1, 3, 5, 7, 9, 11, 13, 15, 17, and 19 on page 296.

ASSIGNMENT 142

Study carefully the rules and explanatory problems given under §159 and §160 and apply the principles involved to the solution of exercises 1, 3, 5, 7, 9, 11, 13, and 15 under §161.

ASSIGNMENT 143

Learn the rule given under §162 and apply it to the solution of the following exercises at the bottom of page 297: 1, 3, 5, 7, 9, 11, 13, 15, 17, and 19.

ASSIGNMENT 144

Sections 163, 164, and 165 require serious study on your part. Your attention is especially directed to the proof under §164. After you understand this proof you will see that any quantity raised to the zero power equals 1.

$$10^0 = 1.$$
$$(a + 1)^0 = 1.$$
$$(a + b + c)^0 = 1.$$

$4x^0 + (3x)^0 + 1^{3x} = 4 + 1 + 1 = 6$. Do you understand that this is so? Why is $1^{3x} = 1$?

Apply the rules of sections 163, 164, and 165 to the solution of exercises 1, 3, 5, 7, 9, 11, 13 and 15.

ASSIGNMENT 145

Solve exercises 17, 19, 21, 23, 25, 27, 29, 31, 33, and 35 under §165.

ASSIGNMENT 146

Solve exercises 1, 3, 5, 8, 13 and 15 under "Supplementary Exercises" on page 299.

ASSIGNMENT 147

Solve problems 1, 4, 6, 9, and 10 on page 300.

ASSIGNMENT 148

Solve problems 13, 18, 22, 24, 26, 30, and 34 on page 301.

ASSIGNMENT 149

Solve problems 36, 39, 41, 44, 47, and 51 on page 302.

ASSIGNMENT 150

Solve problems 53, 55, 57, 59, and 61 on page 303.

GENERAL REVIEW PROBLEMS

(To Be Used As Desired)

1. Simplify $1 - \{1 - [1 - (1 - \overline{1 + x})] - x\} - x$. Ans. $x + 1$.

2. Simplify $\overline{R - 1} - \overline{R + 1} + [R - \overline{1 - R} - \overline{R - 1} - (1 - R) - (R - 1)]$. Ans. $R - 2$.

3. Prove that $[-2 - (-1) + (-5)] + [(-2) + (-1) - (-3)] + [3 - (-2) - (-1)] = 0$.

4. Inclose the last four terms of $u - 8v + 4w - 3x + 7y$ in parentheses preceded by a minus sign.

5. By how much does $2m\ [3m - 2\ (m - \overline{1 + 2m})]$ exceed $(m + 5)\ (4m - 3) + 7$? Ans. $6m^2 - 13m + 8$.

6. Find the sum of $(x + 8)\ (x + 2)$ and $(x - 5)^2$. Ans. $2x^2 + 41$.

7. Subtract $(w - 1)\ (w + 5)$ from $(w + 2)^2$ and add $(w - 3)\ (w + 3)$ to the result. Ans. w^2.

8. From $(r+s-x)a + (r-s+x)c$ subtract $(r+s-x)a - (r+s-x)c$. Ans. $2cr$.

9. Show that $4a^2 + (a + 1)^2 = (a - 1)^2 + (2a + 1)^2 - 1$.

10. Multiply $x^2 + y^2 + z^2 + 2xy - xz - yz$ by $x + y + z$. Ans. $x^3 + 3x^2y + 3xy^2 + y^3 + z^3$.

11. Multiply $\frac{x^2}{2} - \frac{3x}{2} + \frac{1}{2}$ by $\frac{2x^3}{3} + \frac{x^2}{3} - \frac{x}{2} + \frac{1}{4}$.

Ans. $\frac{1}{3}x^5 - \frac{5}{6}x^4 - \frac{5}{12}x^3 + \frac{25}{24}x^2 - \frac{5}{8}x + \frac{1}{8}$

12. Add the quotient of $(R^3 - 1) \div (R - 1)$ to that of $(R^3 - 2R + 1) \div (R - 1)$. Ans. $2R^2 + 2R$.

13. Show that $(1 - 3s + s^2)^2 + s\ (1 - s)\ (2 - s)\ (3 - s) - 1 = 0$.

14. Divide $a^3 - 3ad + d^3 + 1$ by $a^2 - ad - a + d^2 - d + 1$. Ans. $a + d + 1$.

15. By how much does $(m^2n^2 + 3mn + 2)^2$ exceed $2\ (3m^3n^3 + 2m^2n^2 + 6mn)$? Ans. $m^4n^4 + 9m^2n^2 + 4$.

16. Factor $(R + 2)^2 - 7\ (R + 2) + 12$.

17. Factor $(a + x)^2 - 7 - 3\ (a + x + 1)$. Ans. $(a + x - 5)\ (a + x + 2)$.

18. Factor $(m^2 + m - 1)^2 - (m^2 - m - 1)^2$.

19. Factor $cx + dy - dx - cz - cy + dz$. Ans. $(c - d)(x - y - z)$.

20. Factor $(c + d)(x^2 - 1) - (x + 1)(c^2 - d^2)$. Ans. $(x - 1 - c + d)(x + 1)(c + d)$.

21. Factor $(A - 2)(A - 4)(A - 3) - (A - 2) + (A - 2)(A - 3)$. Ans. $(A - 2)^2 (A - 4)$.

22. Factor $M^2 (M - 5)^2 + 2M (M^2 - M - 20) + (M + 4)^2$. Ans. $(M - 2)^4$.

23. Factor $c^3 - 1 - b^2c + b^2$. Ans. $(c - 1)(c^2 + c + 1 - b^2)$.

24. Factor $2 D^4 + 128$. Ans. $2 (D^2 + 4D + 8)(D^2 - 4D + 8)$.

25. Factor $2a^2b^2c^2d - 2abcdmn + 2ab^2cmp - 2bm^2np$. Ans. $2b (abc - mn)(acd + mp)$.

26. Find the H. C. F. of $m^4 - 8m$, $6m^3 - 10m^2 - 4m$, and $(m^2 - 2m)^2$. Ans. $m (m - 2)$.

27. Find H. C. F. of $2P^3 + 5P^2 + 11P + 8$ and $4P^3 - 11P^2 - 25P - 10$.

28. Find L. C. M. of $4R^3 + 4R^2 + 11R - 7$ and $6R^3 + R^2 - 1$. Ans. $(2R - 1)(2R^2 + 3R + 7)(3R^2 + 2R + 1)$.

29. Show that $\left(1 - \frac{12}{x+3} + \frac{4}{x+1}\right)\left(\frac{12}{x-3} + 1 - \frac{4}{x-1}\right) - 1 = 0$.

30. Simplify $\left\{\frac{\frac{N^3}{8} - 1}{\frac{N^3}{8} + 1} \cdot \frac{1 + \frac{4}{N^2} - \frac{2}{N}}{\frac{4}{N^2} + \frac{2}{N} + 1}\right\} \div \left\{\frac{\frac{1}{N} - \frac{1}{2}}{\frac{1}{N} + \frac{1}{2}}\right\}^2$

Ans. $\frac{2 + N}{2 - N}$

31. Solve $\frac{2}{1 - 2y} + \frac{4}{1 - 4y} - \frac{6}{1 - 3y} = 0$. Ans. 0.

32. Prove that $\dfrac{1}{1-\dfrac{1}{MN}} \cdot \dfrac{1}{1+\dfrac{1}{MN}} \div \dfrac{1}{MN-\dfrac{1}{MN}} = MN.$

33. Prove that the sum of any five consecutive numbers equals 5 times the middle one.

34. If $x = 2$, $y = 3$, and $z = -4$, find the value of $(3x - z)(x^2 + z^2)\sqrt{(7x + z)\ (4y - x)}$. Ans. 2000.

35. At what time between 8 and 9 o'clock are the hands of a clock 5 minutes apart? Ans. $38\frac{2}{11}$ min. after 8.

36. What value of m will make $(2m - 1)(6m + 5) = (4m + 3)(3m - 1)$? Ans. -2.

37. By what must $a^4 + 2a^3 + 3a^2 + 4a + 5$ be divided to give a quotient of $a^2 + 4a + 14$ and a remainder of $44a + 47$? Ans. $a^2 - 2a - 3$.

38. Solve for s:
$(s - a)(s + b) - (s + a)(s - b) - 2(a - b) = 0.$
Ans. -1.

39. Solve for n in $l = (n - 1)(a - l) + a$. Ans. 0.

40. Solve $\begin{cases} x + z = 10. \\ y + z = 3. \\ x + y = 9. \end{cases}$ Ans. $x = 8.$ $y = 1.$ $z = 2.$

41. Solve $\begin{cases} a + b + d = 24. \\ a + b + c = 25. \\ a + d + c = 26. \\ b + d + c = 27. \end{cases}$ Ans. $a = 7.$ $b = 8.$ $c = 10.$ $d = 9.$

42. Solve $\begin{cases} u + z + 3y + 2x = 13. \\ 3x + 4y - 2z - u = 15. \\ 4x - 5y + 2u + 3z = 5. \\ 3x - 4z - 5y - 3u = -5. \end{cases}$ Ans. $x = 3.$ $y = 2.$ $z = 1.$ $u = 0.$

43. Solve $\begin{cases} 4 = \dfrac{2y - x - 3}{4} - \dfrac{y - 2x - 3}{3} \\[2ex] 4 = \dfrac{2y - 4x + 9}{3} - \dfrac{4y - 3x - 3}{4} \end{cases}$ Ans. $x = 31.$ $y = -55.$

44. Solve $\begin{cases} \dfrac{x}{m} + \dfrac{y}{n} = 1 + y. \\[2ex] \dfrac{x}{n} + \dfrac{y}{m} = 1 + x. \end{cases}$ Ans. $x = y = \dfrac{mn}{m - mn + n}$

45. Solve $\begin{cases} \dfrac{x}{a+b} + \dfrac{y}{a-b} = 2. \\[2ex] \dfrac{x}{a-b} + \dfrac{y}{a+b} = \dfrac{2(a^2+b^2)}{a^2-b^2} \end{cases}$ Ans. $x = a + b.$ $y = a - b.$

46. Expand $(a^{\frac{1}{2}} - 2\sqrt[3]{a})^6$. Ans. $a^3 - 12a^{\frac{17}{6}} + 60a^{\frac{8}{3}} - 160a^{\frac{5}{2}} + 240a^{\frac{7}{3}} - 192a^{\frac{13}{6}} + 64a^2.$

47. Simplify $\sqrt{\frac{25}{3}} + \frac{1}{3}\sqrt{2} + \sqrt{\frac{8}{9}} - \sqrt{\frac{1}{3}} - \sqrt{8}.$

Ans. $\frac{4}{3}\sqrt{3} - \sqrt{2}.$

48. Solve $\begin{cases} ax - by = 2 - b^2. \\ ay - bx = a^2 - b^2. \end{cases}$ Ans. $x = y = a + b.$

49. Find the square root of $y^{\frac{2}{3}} + y^{\frac{5}{3}} - 4y^{\frac{5}{6}} + 4y + 2y^{\frac{7}{6}} - 4y^{\frac{4}{3}}.$

50. Simplify $\left(\sqrt{\frac{1 - \sqrt{m}}{2}} + \sqrt{\frac{1 + \sqrt{m}}{2}}\right)^2.$ Ans. $1 + \sqrt{1 - m}.$

51. What is the square root of $a(a^3 - 4) + 1 - 2a^2(2a - 3)$?

52. Find the decimal places the value of $\dfrac{3\sqrt{2} + 2\sqrt{3}}{3\sqrt{3} - 2\sqrt{2}}$

Ans. 3.254 +.

53. Is the expression $m^{4y}+2m^{3y}+m^{2y}+2m^{y}+2+m^{-2y}$ a perfect square?

54. If $a^{-\frac{3}{2}} = b^{-1}$ and $b^{\frac{2}{3}} = \frac{4}{9}$, find the numerical value of a.
Ans. $\frac{4}{9}$.

55. Show that $\dfrac{(a^2+b^2)\,b^{-1}-a}{b^{-1}-a^{-1}} \times \dfrac{a^2-b^2}{a^3+b^3} = a.$